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Pre-Calculus

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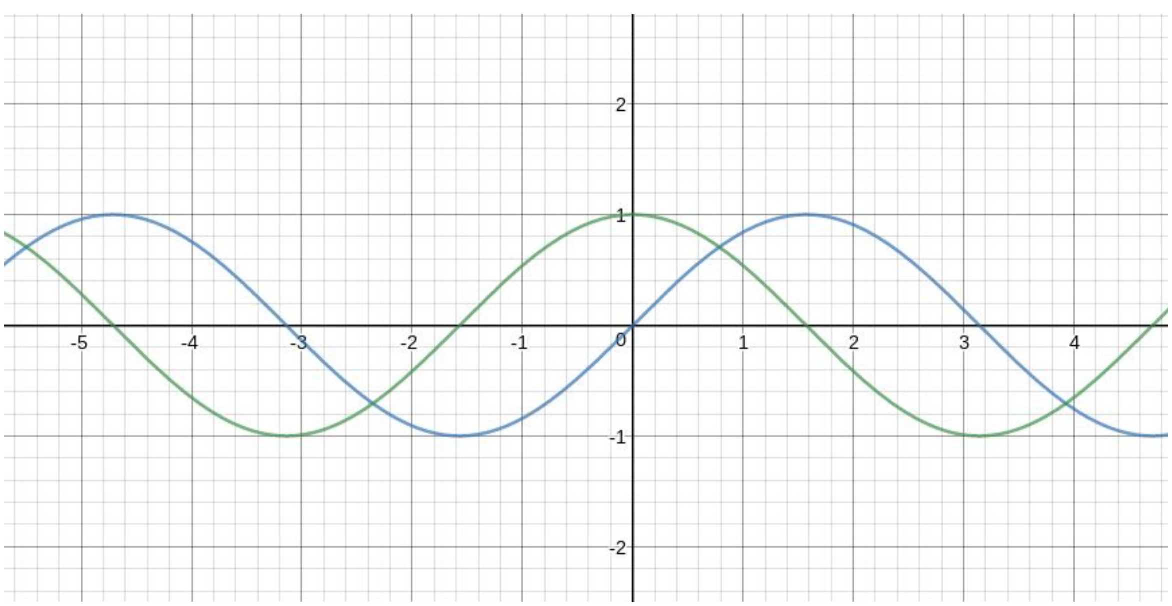
Modeling with Transformations of Trigonometric Functions

Task #1

Periodic functions are functions that repeat forever, returning to the same value at regular intervals. They repeat, identically, over and over as it is followed from left to right or vice versa. Functions like sine and cosine are periodic functions because they go on forever unless given a restriction. Sine and cosine functions also fall in the same subset of sinusoidal functions. A sinusoidal function is a function that is like or similar to a sine function. Sine and cosine are sinusoidal because they are horizontal translations of one another.

$$\sin\theta=\cos(\theta-\frac{\pi}{2}) \text{ and } \cos\theta=\sin(\theta+\frac{\pi}{2})$$



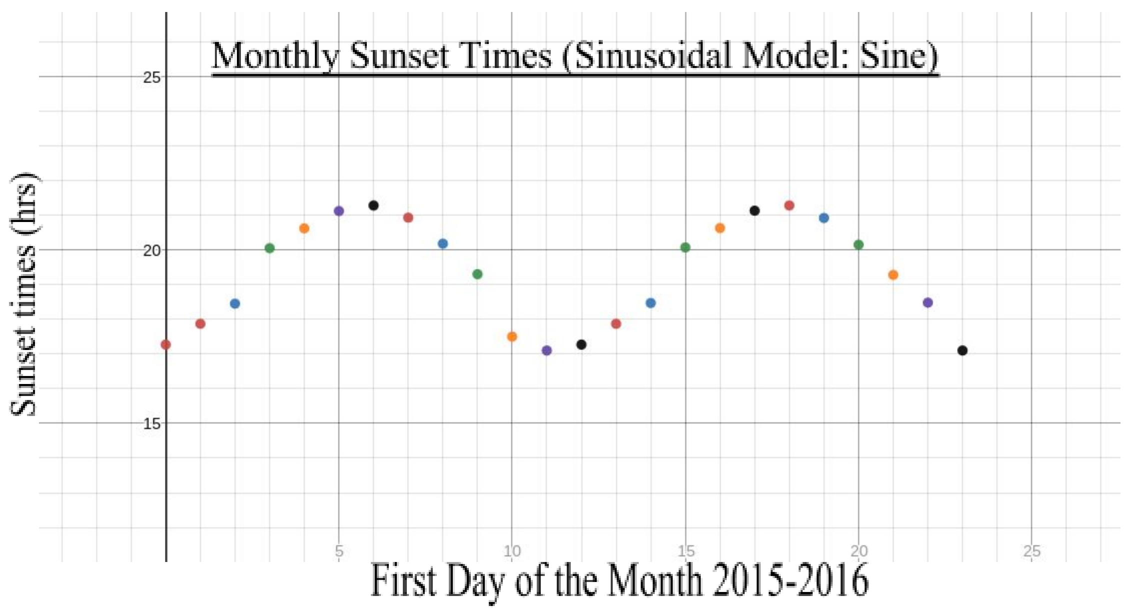
Sine is shown in blue and cosine is shown in green.

My assigned American city is Jackson located in Mississippi. Mississippi is located in the central region of the United States and therefore have a central time zone. I have collected the sunset times in Jackson, MS, for the first day of each month from January 2015 to December 2016. That is over the course of two full years. The data below shows those sunset times for the first day of each of those months. Once I had gathered all of the sunset times for the first day of each month from January 2015 to December 2016, I converted those values from raw hour and minute into military decimal hours.

*The original raw hour and minute times were converted to military decimals by taking the minute value of that time and dividing it by 60, and then adding the hour value to it. Once that was done, I added 12 to it to represent its military value. Values were also rounded to the nearest hundredth if necessary.

First Day of Month	Sunset Time	Independent Variable (x)	Sunset Times in hours ($f(x)$)
January 1, 2015	5:16pm	0	17.27
February 1, 2015	5:52pm	1	17.87
March 1, 2015	6:27pm	2	18.45
April 1, 2015	8:03pm	3	20.05
May 1, 2015	8:37pm	4	20.62
June 1, 2015	9:07pm	5	21.12
July 1, 2015	9:17pm	6	21.28
August 1, 2015	8:56pm	7	20.93
September 1, 2015	8:11pm	8	20.18
October 1, 2015	7:18pm	9	19.30
November 1, 2015	5:30pm	10	17.50

Decmeber 1, 2015	5:06pm	11	17.10
January 1, 2016	5:16pm	12	17.27
February 1, 2016	5:52pm	13	17.87
March 1, 2016	6:28pm	14	18.47
April 1, 2016	8:04pm	15	20.07
May 1, 2016	8:38pm	16	20.63
June 1, 2016	9:08pm	17	21.13
July 1, 2016	9:17pm	18	21.28
August 1, 2016	8:55pm	19	20.92
September 1, 2016	8:09pm	20	20.15
October 1, 2016	7:17pm	21	19.28
November 1, 2016	6:29pm	*22*	18.48
December 1, 2016	5:06pm	23	17.10



The plotted data shown is almost a perfect replica of a sinusoidal function. To model its proximity, I must first generate an equation for it. I will generate this equation using only the data that I have collected and what is shown. Because I was assigned to represent my equation using the sin function, I can assume that my equation will be generated in the format below.

$$f(x) = A\sin(Bx - C) + D$$

Here, x is the independent variable, representing the first day of each month. $f(x)$ is the dependent variable, and A, B, C, and D are real number parameters.

I will start off finding the value of D. I'm starting off with finding the value of D because it is a simple task. To find the value of D, I must find the greatest plotted $f(x)$ value and add it to the lowest, then divide it by 2.

$$D = \frac{y\max + y\min}{2}$$

$= 21.28+17.10 \rightarrow 38.38$

$= \dfrac{38.38}{2}$

$D = 19.19$

Next, I will find the value of A. The steps to finding the value of A is similar to the steps of finding D. I must find the absolute value of the greatest plotted $f(x)$ value and subtract the lowest from it. I must then divide it by 2.

$$A = \frac{|y\max - y\min|}{2}$$

$= |21.28-17.10| \rightarrow |4.18|$

$= \dfrac{4.18}{2}$

$A = 2.09$

Next, I will find the value of B. Because I am using the sine function, I know that the period will be $\frac{2\pi}{|B|}$.

$$\frac{2\pi}{|B|} = 11$$

The period equals 12 because that's how many months passed for lowest $f(x)$ value to be repeated again.

$$|B| \times \frac{2\pi}{|B|} = 12 \times |B|$$

$$\frac{2\pi}{12} = \frac{12|B|}{12}$$

$$\frac{2\pi}{12} = |B|$$

$\pm \frac{\pi}{6} = B$ *B can either equal $\frac{\pi}{6}$ or - $\frac{\pi}{6}$.

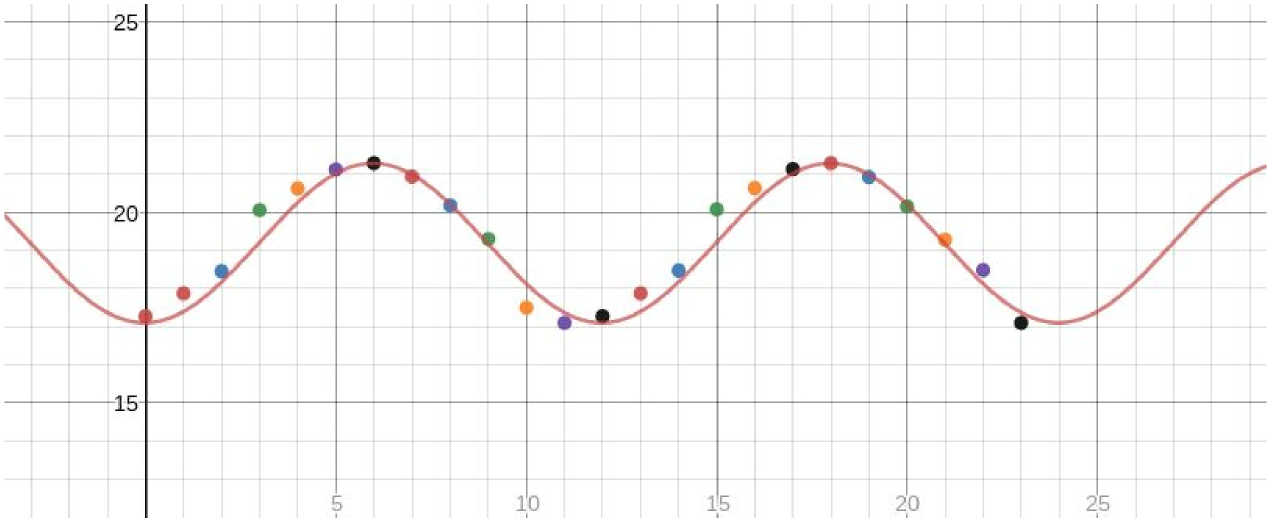
Lastly, I will find the value of C. Since the phase shift is equivalent to $\frac{C}{B}$, I can use the B value to find C. $\frac{C}{\frac{\pi}{6}}$ has to equal a value greater than 2 and less than 3 because there is no plotted value of x that has a $f(x)$ value of 19.19. 2 has a value of 18.45 and 3 has a value of 20.05, so a number that has a $f(x)$ value of 19.19 must lie between those two numbers. Looking at the graph above, I can see that the actual function will cross the midline at 3.026, but since I already have a collected data point at 3 that is greater than 19.19, I will subtract 0.026 from 3 instead.

$$\frac{C}{\frac{\pi}{6}} = 2.974$$

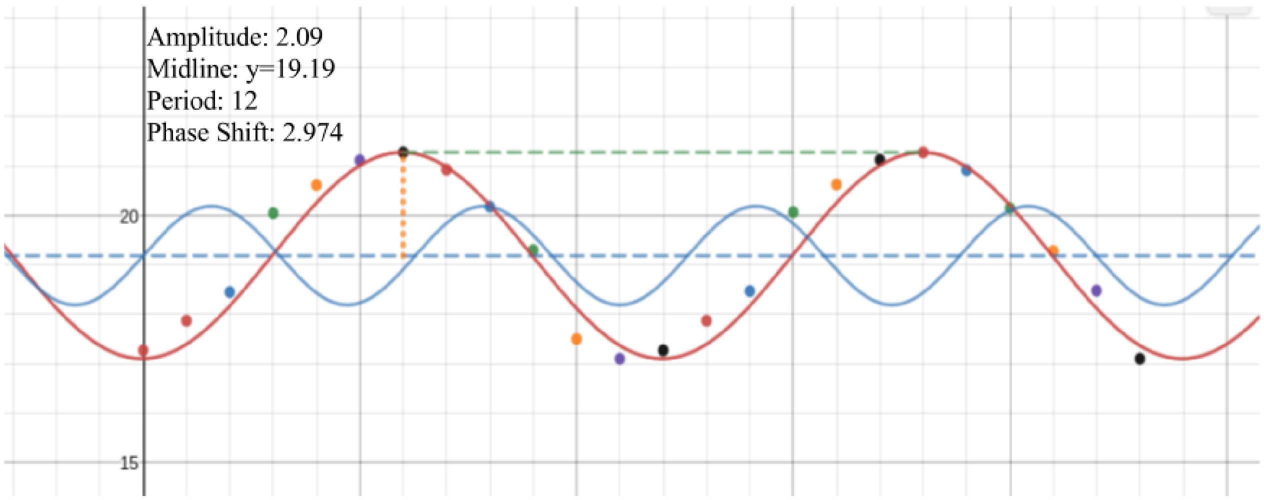
$$\frac{\pi}{6} \times \frac{C}{\frac{\pi}{6}} = 2.974 \times \frac{\pi}{6}$$

$$C = \frac{2.974\pi}{6}$$

Putting all found parameters into the beginning formated equation, the equation for my function should be... $f(x) = 2.09\sin(\frac{\pi}{6}x - \frac{2.974\pi}{6}) + 19.19$



The parameter *A* represents the amplitude in this sinusoidal function. The amplitude is the distance from the highest point to the midline of the function. The amplitude is shown as the orange dotted line in the picture below. The parameter *D* represents the midline of the function. The midline is a horizontal axis that divides the function in half. It has an equal distance from the highest *y* value and the lowest *y* value. The midline is shown as the blue dashed line in the picture below. The parameter *B* represents the horizontal dilation of the function. It compresses a curve when the absolute value of *B* is greater than 1 and stretches the curve when the absolute value of *B* is greater than 0 and less than 1. The absolute value of *B* also influences the period of the function. The period of the function is the horizontal distance before the function's behavior repeats. The period is shown as the green dashed line in the picture below. The parameter *C* represents the part of the phase shift. The value of $\frac{C}{B}$ gives the phase shift of the function. The phase shift is the horizontal translation from the parent sine function. The parent sine function is shown as the blue line in the picture below.



*Note: to compare the parent sin function to the generated function, I had to increase its midline to 19.19. The original midline of the parent sin function is *y*=0.

Because the data that I collected was a little irregular, I had to make the error of estimating a few things so that the equation's graph would fit the data better. I understand that the work may not be 100 percent accurate, but it does demonstrate my knowledge of finding an equation based off of a given set of data.

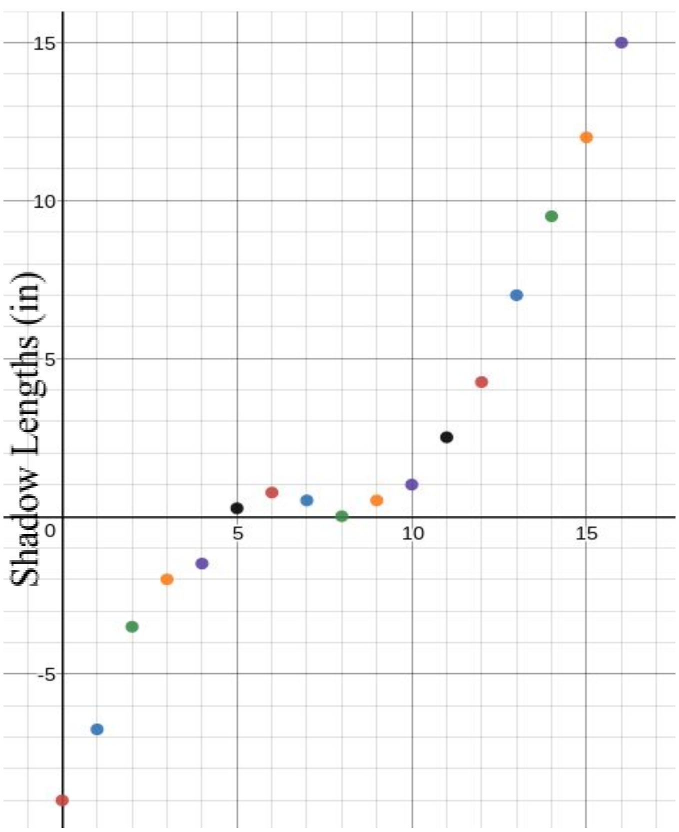
Task #2

For this part of the project, I was required to go outside and physically collect data. I had to make a sundial using a writing utensil and a flat surface. Once the sundial was made and functioning, I had to collect measurements of shadow lengths every half an hour. I had to do this from 8 am to 4 pm. Once I had collected all 17 measurements, I was able to make a table displaying the data. My independent variable, x, will represent my time intervals when plotting coordinates. When organizing and graphing my data, $x=0$ corresponds to 8 am. So for the 17 data points that I have, the independent variable, x, will be values 0 to 16.

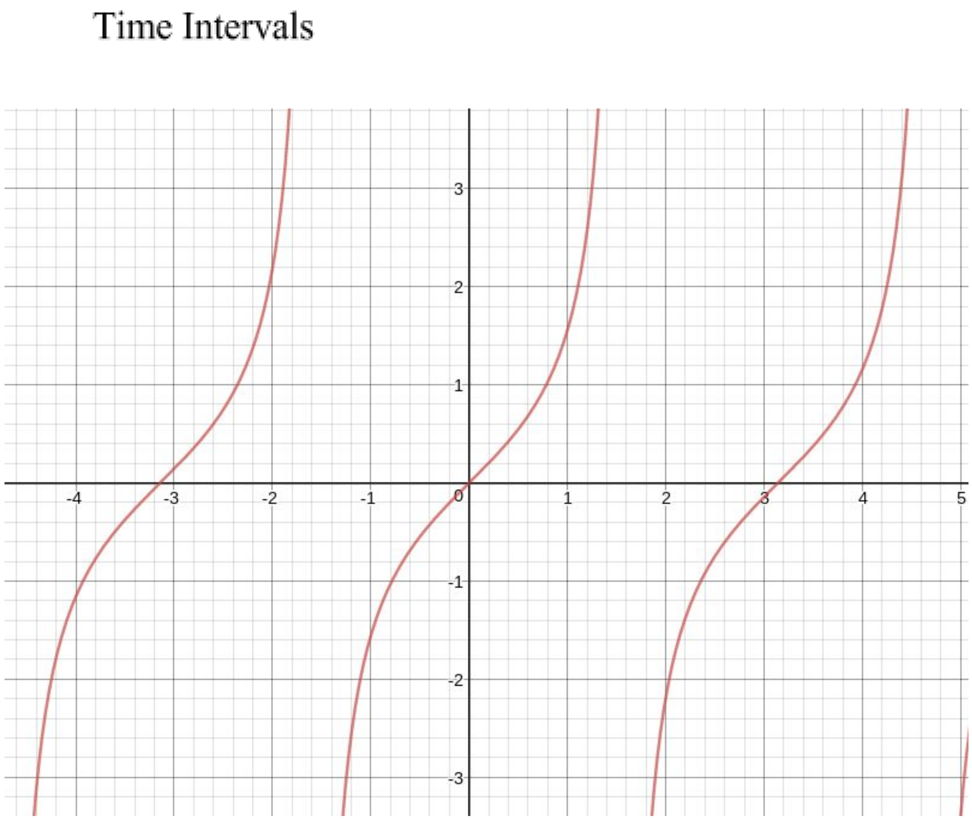
I then converted values of the dependent variables from raw lengths into lengths with respect to the shadow length at noon. Since 12 pm is the median for the data, it will represent the base length. This means that the length found before noon should be negative with respect to the baseline, and lengths found after noon should be positive, still with respect to the baseline. I did this by subtracting dependent values from the baseline's dependent value. Once I did all values before the baseline, I continued for values after the baseline, but with a minor difference. I subtracted the baseline's dependent value from every dependent value after it. After I had to convert my shadow lengths based on their position, I was able to plot the data on a graph.

Time Intervals	Shadow Measurements (inches)	Independent Variable (x)	Converted Lengths $F(x)$
8:00 am	17 inches	0	-9
8:30 am	14.75 inches	1	-6.75
9:00 am	11.5 inches	2	-3.5
9:30 am	10 inches	3	-2
10:00 am	9.5 inches	4	-1.5

10:30 am	7.75 inches	5	0.25
11:00 am	7.25 inches	6	0.75
11:30 am	7.5 inches	7	0.5
12:00 pm	8 inches	8	0
12:30 pm	8.5 inches	9	0.5
1:00 pm	9 inches	10	1
1:30 pm	10.5 inches	11	2.5
2:00 pm	12.25 inches	12	4.25
2:30 pm	15 inches	13	7
3:00 pm	17.5 inches	14	9.5
3:30 pm	20 inches	15	12
4:00 pm	23 inches	16	15



This graph looks different than the ones of *task #1*. That is because this graph is not a sinusoidal function. This is a tangent function. Although this graph is still periodical, it is not sinusoidal. A cotangent function is just like a tangent one. Cotangent is just the reciprocal of tangent, and when graphed they are reflections of one another, either over the *x* or *y* axis. Below is a parent tangent function.



The data that I had collected did not form the best tangent function. I am not aware of where my data went wrong, if it did at all. I will still be able to generate an equation that will fit my data in some way.

Because I was assigned to use the tangent function, I know the format of my equation will be…

$$F(x) = A\tan(Bx - C) + D$$

The steps to finding the parameters to generate my equation will be very similar to those of task #1. I will be able to find every parameter except for *A*. I will not be able to find *A*, unless I plug in a coordinate and solve. I will only be able to do so after I find the other three parameter. The reasoning for this is because tangent functions have an undefined amplitude. The equation that I used to find *A*, in task #1, was also an equation to define the amplitude. But since my function will have an undefined amplitude, the equation will not work.

First, I will find the *B* value. Because I am using the tangent function, I know that the period will be $\frac{\pi}{|B|}$.

$$\frac{\pi}{|B|} = 16$$

The period is set equal to 16 because that's how many time intervals passed for the opposite *F*(*x*) value to be repeated again.

$$|B| \times \frac{\pi}{|B|} = 16 \times |B|$$

$$\pi = 16|B|$$

$$|B| = \frac{\pi}{16}$$

$B = \pm \frac{\pi}{16}$ **B* can either equal $\frac{\pi}{16}$ or - $\frac{\pi}{16}$.

Next, I will find *D*. It is not complicated at all to find the value of *D* because middle of the function is already defined at noon which has a dependent value of 0. Therefore, *D*=0.

Lastly, I will find *C*. Since the phase shift is equivalent to $\frac{C}{B}$, I can use the *B* value to find *C*.

$$\frac{C}{\frac{\pi}{16}} = 8$$

The phase shift is set equal to 8 because that is the midline is happening at *x*=8.

$$\frac{\pi}{16} \times \frac{C}{\frac{\pi}{16}} = \times \frac{\pi}{16}$$

$C = \frac{\pi}{2}$

Now that I have 3 out of 4 parameter, I can plug in a plotted coordinate into the formated tangent equation.

$4.25 = A\tan(\frac{\pi}{16}\,(12) - \frac{\pi}{2}) + 0$

$4.25 = A\tan(\frac{3\pi}{4} - \frac{\pi}{2})$

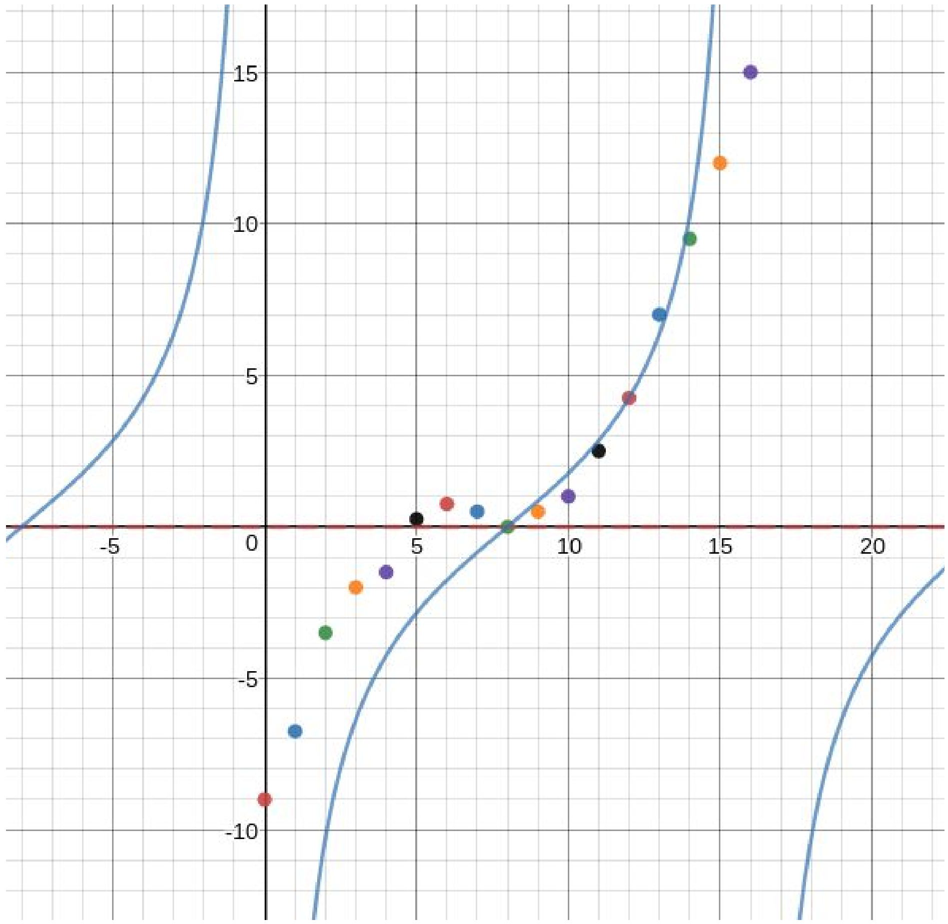
$4.25 = A\tan(\frac{\pi}{4})$

$4.25 = A(1)$

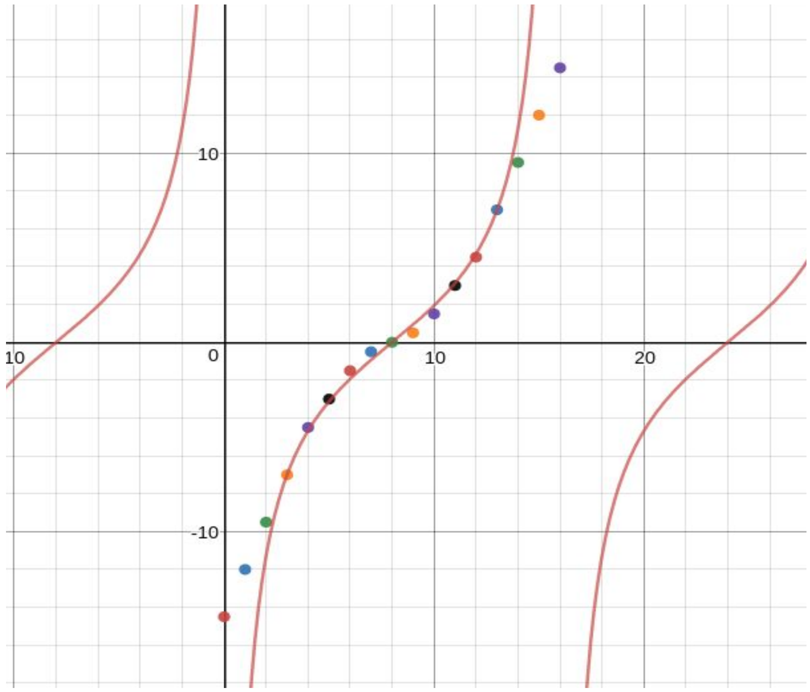
$A = 4.25$

Now that I have all parameters, I can generate an equation and graph it.

$$F(x) = 4.25\tan(\frac{\pi}{16}x - \frac{\pi}{2})$$

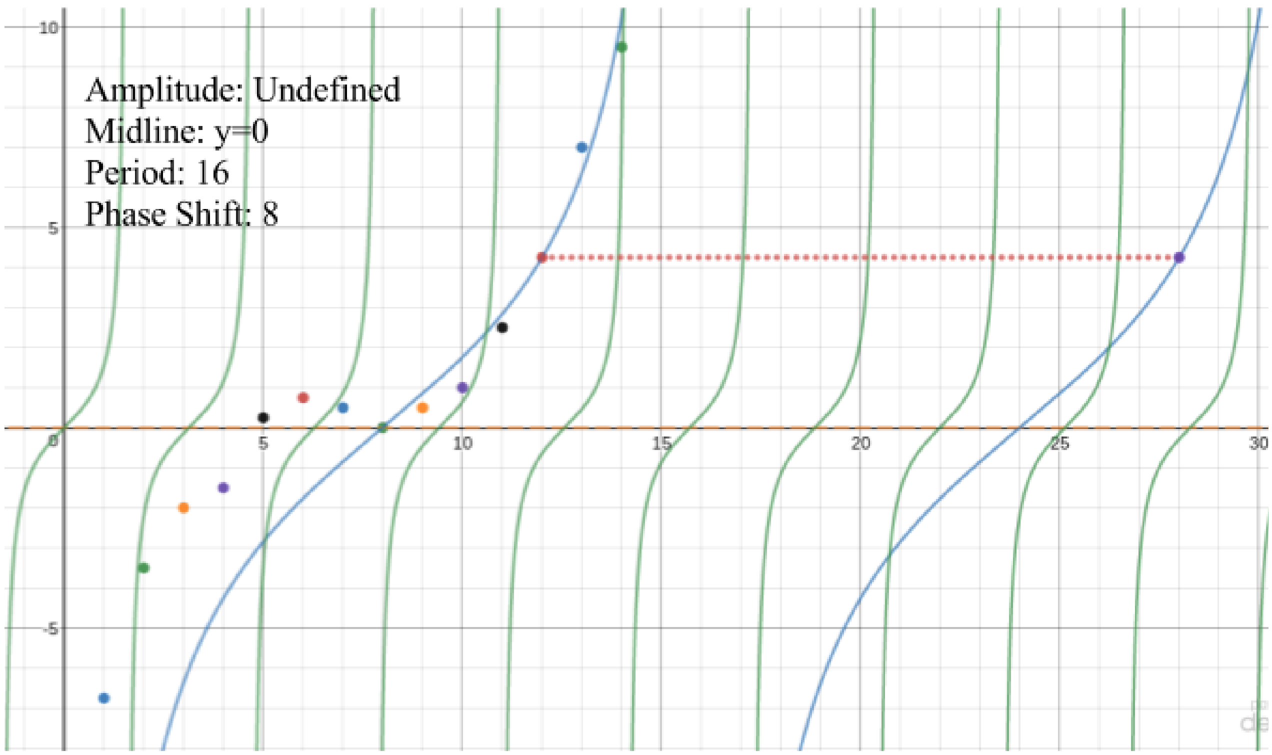


Just as I assumed, I was able to generate an equation fitting some of my data, but not entirely all. I would assume that this is due to some type of error. There must have been an error in my data somehow. Maybe it happened when it was collected outside. Since my collected data is irregular my generated function does not fit it perfectly. It is aligned with the positive portion of my data, but not the negative. Below I have inserted an example graph. This graph displays (almost) perfect data collected from a virtual sundial. When using all of the steps that I used to generate a function for my data, I was able to create an almost perfect equation representing it. This shows that the error of my irregular data is the reason that my generated equation is inaccurate.



In this case, the parameter *A* represents the vertical dilation in this tangent function. *A* compresses the function's curve when its absolute value is greater than 0, but less than 1. It also stretches the curve when its absolute value is greater than 1, and when *A* is less than 0 it reflects over the *x* axis. As said before, the amplitude is undefined for this type of function. This is due to the fact that this function has an infinite range. The parameter *D* represents the midline of the function. The midline is shown as the orange dashed line in the picture below. The parameter *B* represents the horizontal dilation of the function. The absolute value of *B* also influences the period of the function. The period of the function is the horizontal distance before the function's behavior repeats. The period is shown as the red dotted line in the picture below. The parameter

C represents the part of the phase shift. The value of $\frac{C}{B}$ gives the phase shift of the function. The phase shift is the horizontal translation from the parent tangent function. The parent tangent function is shown as the green line(s) in the picture below.



Doing this project I am now able to understand how sinusoidal and other periodic functions apply to the real world. When learning new material, I usually wonder "what are some real life examples of this?". And when I try to think or imagine some examples, it never works out for me. This project was fully based off of collecting data from actual things that portray to our everyday lives. I never would have suspected that collecting data from a sundial would have created a tangent/cotangent function. The same goes for sunset times over multiple years. This project has made me aware that finding these function applied in everyday life is more simple than one would think.

This all matters because this indicates that accurate predictions can be made to determine future sun positions. Questions are being answered for people who wonder how meteorologists can predict the weather, sunset time, the Earth's position, and etc. If periodic and sinusoidal functions can be represented through sun position related matter, than what else can it represent? Overall the biggest output from this is basically understanding how the future can be determined. Based on the parent function's repetitive nature, it is almost always a certainty to know what happens next.

Work Cited

Stapel, Elizabeth. "Trigonometric Functions and Their Graphs: Tangent." Purplemath. Accessed 15

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